(11) Q.No. → If $x = \sin(\log y)$, prove that $(1-x^2)y_2 - xy_1 = y$

Ans. → $\because x = \sin(\log y)$

$\log y = \sin^{-1}x$

D.b.s. w.r.t. x, we have

$\frac{1}{y} \times y_1 = \frac{1}{\sqrt{1-x^2}}$

$y_1\sqrt{1-x^2} = y$

Squaring both sides, we have

$y_1^2(1-x^2) = y^2$

Again, D.b.s. w.r.t. x, we have

$2y_1 \cdot y_2(1-x^2) + y_1^2 \times -2x = 2yy_1$

$y_2(1-x^2) - xy_1 = y$ Proved.

(12) Q.No. → If $y = (\tan^{-1}x)^2$, prove that $(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2$.

Ans. → $\because y = (\tan^{-1}x)^2$

D.b.s. w.r.t. x, we have

$y_1 = 2\tan^{-1}x \times \frac{1}{1+x^2}$

or, $y_1(1+x^2) = 2\tan^{-1}x$

$(1+x^2)y_1 = 2\tan^{-1}x$

Again diff. b.s. w.r.t. x, we have

$(1+x^2)y_2 + y_1 \times 2x = \frac{2}{1+x^2}$

or, $(1+x^2)y_2 + 2x(1+x^2)y_1 = 2$ Proved.

(13) Q.No → If $y = \cos(m\sin^{-1}x)$, prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$.

Ans. → $\because y = \cos(m\sin^{-1}x)$

D.b.s. w.r.t. x, we have

$y_1 = -\sin(m\sin^{-1}x) \times m \times \frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\, y_1 = -m\sin(m\sin^{-1}x)$

$\sqrt{1-x^2}\cdot y_1 = m\sqrt{1-\cos^2(m\sin^{-1}x)}$

or, $\sqrt{1-x^2}\, y_1 = m\sqrt{1-y^2}$

Squaring both sides, we have

or, $(1-x^2)y_1^2 = m^2(1-y^2)$

D.b.s. w.r.t. x, we have

$(1-x^2)\cdot 2y_1\cdot y_2 + -2x\cdot y_1^2 = m^2\cdot -2y\cdot y_1$

$(1-x^2)2y_1y_2 - 2xy_1^2 = -m^2 2y\cdot y_1$

$(1-x^2)y_2 - xy_1 + m^2y = 0$ Proved.

(14) QN. → If $x = \cosh\left(\frac{\log y}{m}\right)$, prove that

$$(x^2-1)y_2 + xy_1 - m^2y = 0.$$

Ans. → $\frac{\log y}{m} = \cosh^{-1}x$

$\log y = m\cosh^{-1}x$

D.b.S.w.r.t. x, we have,

$\frac{1}{y} \cdot y_1 = \frac{m}{\sqrt{x^2-1}}$

$y_1\sqrt{x^2-1} = my$

Squaring both sides, we have

$y_1^2(x^2-1) = m^2y^2$

Again D.b.S.w.r.t. x, we have,

$2y_1 \cdot y_2(x^2-1) + y_1^2 \times 2x = m^2 \times 2yy_1$

$y_2(x^2-1) + xy_1 = m^2y$

$y_2(x^2-1) + xy_1 - m^2y = 0$ Proved

(15) QN. → If $x = \sinh\left(\frac{1}{m}\log y\right)$, Prove that $(x^2+1)y_2 + xy_1 = m^2y$.

Ans. → $\because x = \sinh\left(\frac{1}{m}\log y\right)$

$\therefore \left(\frac{1}{m}\log y\right) = \sin^{-1}hx$

$\log y = m\sin^{-1}hx$

D.b.S. w.r.t. x, we have,

$$\frac{1}{y}\cdot y_1 = \frac{m}{\sqrt{1+x^2}}$$

$$y_1\sqrt{1+x^2} = my$$

Squaring both sides, we have

$$y_1^2(1+x^2) = m^2y^2$$

Again D.b.S. w.r.t. x, we have

$$2y_1\cdot y_2(1+x^2) + y_1^2\times 2x = m^2 \times 2yy_1$$

$$y_2(1+x^2) + xy_1 = m^2y. \text{ proved.}$$

(16) Q No.→ If $y = e^{\cos^{-1}x}$, prove that

$$(1-x^2)y_2 - xy_1 - y = 0$$

Ans.→ $\because y = e^{\cos^{-1}x}$

D.b.S. w.r.t. x, we have

$$y_1 = e^{\cos^{-1}x}\times\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

~~or, $\sqrt{1-x^2}\,y_1 = -y$~~

$$y_1 = y\times\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

or, $\sqrt{1-x^2} \cdot y_1 = -y$

squaring both sides,

$(1-x^2) \cdot y_1^2 = y^2$

Again, D.b.S.w.r.t. x, we have

$(1-x^2) \cdot 2y_1 \cdot y_2 + y_1^2 \times -2x = 2y \cdot y_1$

$2y_1\left[(1-x^2) \cdot y_2 - xy_1\right] = 2y_1 \cdot y$

$(1-x^2) \cdot y_2 - xy_1 = y$

or, $(1-x^2) \cdot y_2 - xy_1 - y = 0$ proved.

(17) QN.→ If $y = Ae^{-Kt}\cos(pt+c)$, Prove that

$$\frac{d^2y}{dt^2} + 2K\frac{dy}{dt} + (p^2+K^2)y = 0$$

Ans.→ $\because y = Ae^{-Kt}\cos(pt+c)$

D.b.S.w.r.t. t, we have

$$\frac{dy}{dt} = A\left[-Ke^{-Kt} \cdot \cos(pt+c) + e^{-Kt} \times -\sin(pt+c) \times p\right]$$

$$= A\left[-Ke^{-Kt} \cdot \cos(pt+c) + pe^{-Kt}\sin(pt+c)\right]$$

$$\frac{dy}{dt} = A \cdot -Ke^{-Kt} \cdot \cos(pt+c) - Ape^{-Kt}\sin(pt+c)$$

$$= -KAe^{-Kt}\cos(pt+c) - Ape^{-Kt}\sin(pt+c)$$

$$\frac{dy}{dt} = -ky - Ap e^{-kt} \sin(pt+c) \quad \text{—} \; (1)$$

$$Ap e^{-kt} \sin(pt+c) = -ky - \frac{dy}{dt} \quad \text{—} \; (a)$$

Again diff. (1) b.s.w.r.t. t, we have

$$\frac{d^2y}{dt^2} = -k\frac{dy}{dt} - Ap\left[-k e^{-kt} \sin(pt+c) + e^{-kt} \cos(pt+c) \times p\right]$$

$$\text{or, } \frac{d^2y}{dt^2} = -k\frac{dy}{dt} + Akp e^{-kt} \sin(pt+c) - Ap^2 e^{-kt} \cos(pt+c)$$

$$\frac{d^2y}{dt^2} = -k\frac{dy}{dt} + Akp e^{-kt} \sin(pt+c) - p^2 A e^{-kt} \cos(pt+c)$$

$$\frac{d^2y}{dt^2} = -k\frac{dy}{dt} + k(Ap e^{-kt} \sin(pt+c)) - p^2 y$$

$$\text{or, } \frac{d^2y}{dt^2} = -k\frac{dy}{dt} + k\left(-ky - \frac{dy}{dt}\right) - p^2 y$$

$$\text{or, } \frac{d^2y}{dt^2} = -k\frac{dy}{dt} - k^2 y - k\frac{dy}{dt} - p^2 y$$

$$\text{or, } \frac{d^2y}{dt^2} = -2k\frac{dy}{dt} - y(k^2+p^2)$$

$$\text{or, } \frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + (k^2+p^2)y = 0 \text{ Proved.}$$